7.5 Finding Angles and Solving Right Triangles

NEW SKILLS: WORKING WITH INVERSE TRIGONOMETRIC RATIOS

The trigonometric ratios discussed in this chapter are unaffected by the size of the triangle, provided that the acute angle remains the same.

If you know the trigonometric ratio, you can calculate the size of the angle. This requires an "inverse" operation. You can use your calculator to find the opposite of the usual ratio calculation. You can think of the inverse in terms of subtraction and addition: subtraction is the inverse, or opposite, of addition because it "undoes" the operation.

For more details, see page 307 of *MathWorks 10.*

Example 1

Calculate each angle to the nearest degree.

a) $\sin A = 0.2546$

b) $\cos B = 0.1598$

c) $\tan C = 3.2785$

SOLUTION

Use the inverse function on your calculator.

a) $\sin A = 0.2546$

$A = \sin^{-1}(0.2546)$

$A \approx 14.7$

$\angle A$ is approximately 15°.

b) $\cos B = 0.1598$

$B = \cos^{-1}(0.1598)$

$B \approx 80.8$

$\angle B$ is approximately 81°.

c) $\tan C = 3.2785$

$C = \tan^{-1}(3.2785)$

$C \approx 73.0$

$\angle C$ is approximately 73°.

BUILD YOUR SKILLS

1. Calculate the angle to the nearest degree.

   a) $\sin D = 0.5491$

   b) $\cos F = 0.8964$

   c) $\tan G = 2.3548$

   d) $\sin H = 0.9998$

2. In right triangle ΔXYZ, the ratio of the side opposite $\angle X$ to the hypotenuse is $\frac{7}{8}$. What is the approximate size of $\angle X$?

When solving this problem on your calculator, put brackets around $\frac{7}{8}$

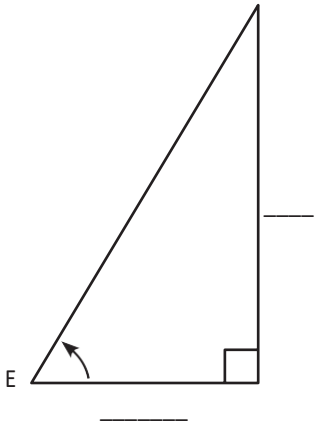
3. What is the approximate size of an angle in a right triangle if the ratio of the side opposite the angle to the side adjacent to the angle is $\frac{15}{8}$?

Example 2

Determine the angle of elevation to the top of a 5-metre tree at a point 3 metres from the base of the tree.

SOLUTION

Sketch a diagram.



When solving this problem on your calculator, put brackets around $\frac{5}{3}$

You are given the height (h, 5 metres) and the length (ℓ, 3 metres) of the triangle, and you need to solve for the angle of elevation. Use the tangent ratio.

$$\tan E = \frac{\text{opp}}{\text{adj}}$$

$$\tan E = \frac{5}{3}$$ Substitute the known values.

$$E = \tan^{-1}\left(\frac{5}{3}\right)$$ Use the inverse function to solve for E.

$E \approx$ _____

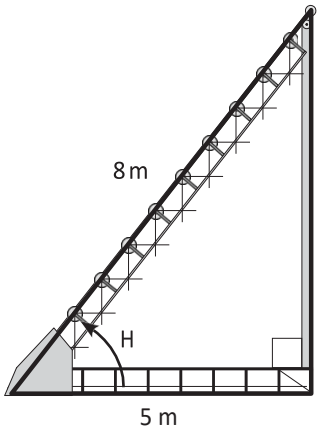
The angle of elevation is approximately _____°.

BUILD YOUR SKILL

4. What is the angle of depression from the top of a 65-metre cliff to an object 48 metres from its base?

5. At what angle to the ground must you place a support if it is 6.8 metres long and must reach 4.2 metres up the side of a tower?

6. At what angle to the ground is an 8-metre long conveyor belt if it is fastened 5 metres from the base of a loading ramp?

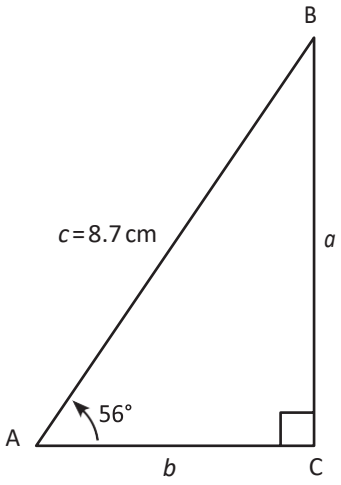


NEW SKILLS: WORKING WITH RATIOS TO SOLVE TRIANGLES

Solving a triangle means finding the values of all the unknown sides and angles. In a right triangle, you already know that one angle is 90°, so there are only five other parts to consider: the three sides, and the two other angles. If you are given any two sides, or any one side and one angle, you can use trigonometry to find the other values.

Example 3

Solve the right triangle. Give lengths to the nearest tenth.



SOLUTION

You are given two of the three angles, so you can solve for the third angle.

$\angle B = 180° - 90° - 56°$

$\angle B = 34°$

To solve for side a, you can use the sine ratio. Use $\angle$A, and the length of the hypotenuse, c.

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\sin A = \frac{a}{c}$$

$$\sin 56° = \frac{a}{8.7}$$ Substitute the known values.

$$\_\_\_\_\_ \approx a$$

Side a is approximately 7.2 cm long.

To solve for b, use the cosine ratio.

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$

$$\cos A = \frac{b}{c}$$

$$\cos 56° = \frac{b}{8.7}$$ Substitute the known values.

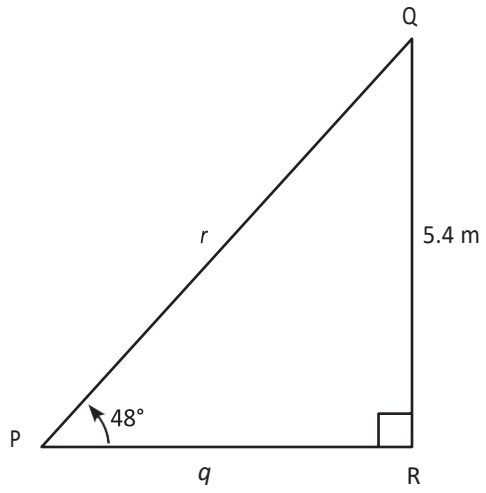
_____ $\approx b$

Side b is approximately _____ cm long.

You could have used the Pythagorean theorem to find side b, but this would have been less accurate because you would have used an approximation for side a. It is always better to use the numbers given, if possible, rather than one you calculated.

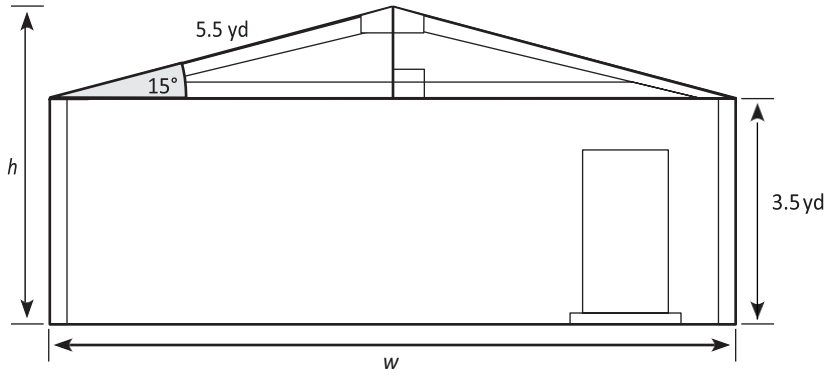
BUILD YOUR SKILLS

7. Solve the given triangle without using the Pythagorean theorem.



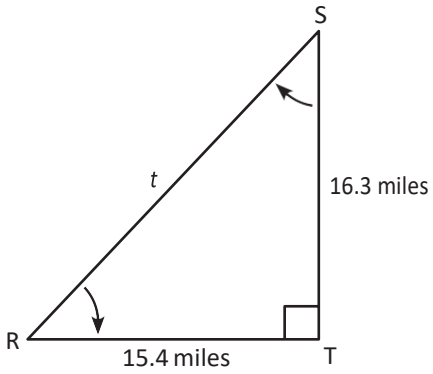
8. The two equal angles of an isosceles triangle are each 70°. Determine the measures of the rest of the triangle if it has a height of 16 cm.

9. The length of the rafter is 5.5 yards, and the side height of the building is 3.5 yards. Determine the width of the building and its total height.



Example 4

Solve the given triangle.



SOLUTION

Calculate $\angle R$ using the tangent ratio.

$$\tan R = \frac{\text{opp}}{\text{adj}}$$

$$\tan R = \frac{ST}{TR}$$

$$\tan R = \frac{16.3}{15.4}$$ Substitute the known values.

$$\angle R = \tan^{-1}\left(\frac{16.3}{15.4}\right)$$ Use the inverse function to solve for $\angle R$

$\angle R \approx$ _____

$\angle R$ is approximately _____.

Calculate $\angle S$ using the measures of the angles in the triangle.

$\angle S = 180° - 90° -$ _____°

$\angle S =$ _____

Calculate the length of side t using the Pythagorean theorem.

$$r^2 + s^2 = t^2$$

$$16.3^2 + 15.4^2 = t^2$$

$$\sqrt{16.3^2 + 15.4^2} = t$$

_____ $\approx t$

Side t is approximately _____miles long.

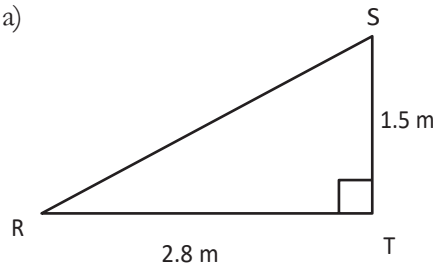
You could have found $\angle S$ first using the tangent ratio.

You could have solved for side t using the sine or cosine functions, but this would have involved rounding errors.

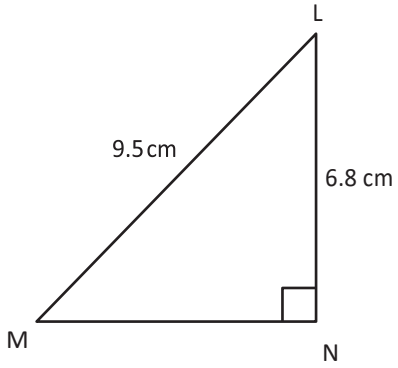
BUILD YOUR SKILL

10. Solve the following triangles.

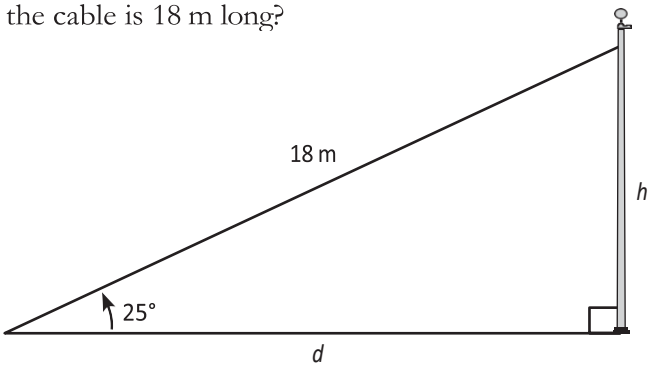
a)



b)

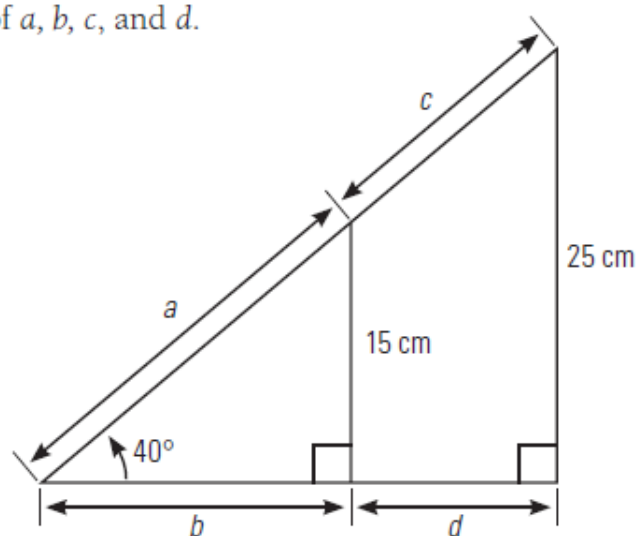


11. What height is a pole, and how far away from it is a cable attached to the ground, if the angle of elevation is 25° and the cable is 18 m long?



12.

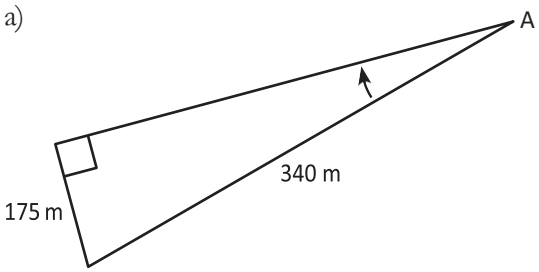
Find the values of a, b, c, and d.



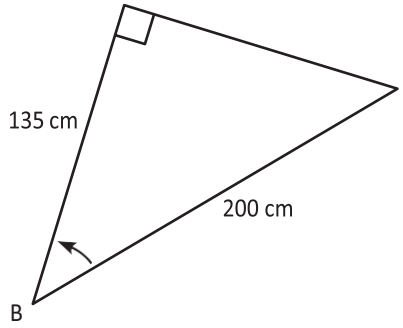
PRACTISE YOUR NEW SKILLS

1. Find the indicated angle in each of the following diagrams.

a)



b)



2. In a right triangle, one acute angle is 22° and the hypotenuse is 70 cm. Find the lengths of the legs and the other angle measure.

3. What is the angle of elevation if a ramp with a height of 1 metre and a horizontal length of 3 metres?

4. A grain auger is 25 feet long. The largest angle of elevation at which it can safely be used is 75°. What is the maximum height to which it can reach and how far from the base of the granary will it be, assuming that it dumps right at the edge?

5. Maura's driveway has an angle of depression of 40° from the flat roadway. If it levels off to the garage floor, which is 3 metres below the roadway, how long is the driveway and how far into the lot is the garage entrance?

6. If a boat is 150 metres from the base of a cliff that is 90 metres high, what is the angle of elevation from the boat to the cliff top?