

## 7.3

## The Cosine Ratio

## NEW SKILLS: WORKING WITH THE COSINE RATIO TO SOLVE TRIANGLES

**cosine ratio:** in a right triangle, the ratio of the length of the side adjacent a given angle to the length of the

hypotenuse (abbreviated as cos)

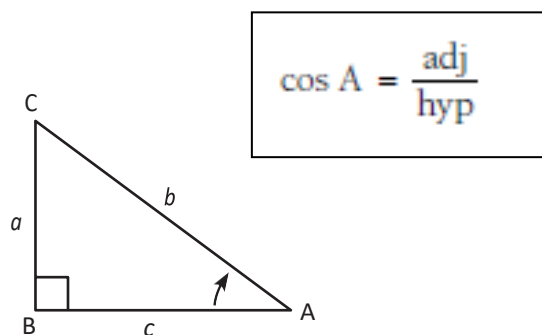
Another important trigonometric ratio of right triangles is the ratio of the side adjacent to the given acute angle to the hypotenuse. This is called the **cosine ratio**.

For a given angle  $A$ , the cosine ratio can be stated as follows.

$$\text{cosine } \angle A = \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}$$

cosine

This ratio can be abbreviated as follows.



For triangle ABC, the cosine of  $\angle A$  can be stated as the following.

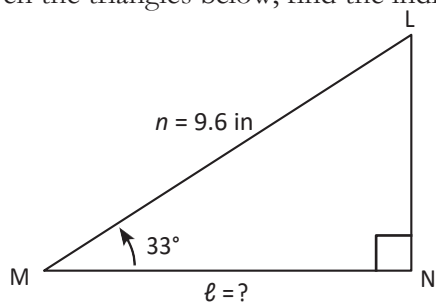
$$\cos A = \frac{c}{b}$$

For more details, see page 293 of *MathWorks 10*.

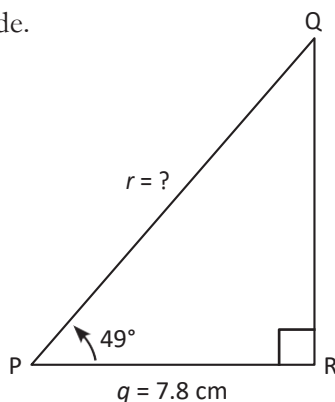
**Example 1**

Given the triangles below, find the indicated side.

a)



b)

**SOLUTION**

a) Use the cosine ratio to solve for  $l$ .

$$\cos M = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 33^\circ = \frac{l}{9.6} \quad \text{Substitute the known values.}$$

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$$\underline{\hspace{2cm}} = l$$

The notation  $9.6 \cos 33^\circ$ , without the multiplication symbol, can be used to mean  $9.6 \times \cos 33^\circ$ .

**ALTERNATIVE SOLUTION**

a) Since you know that the sum of the angles of a triangle is  $180^\circ$ , you can calculate angle L and then use the sine ratio to solve for  $l$ .

$$180^\circ - 90^\circ - 33^\circ = 57^\circ$$

$$\sin L = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 57^\circ = \frac{l}{9.6} \quad \text{Substitute the known values.}$$

$$9.6 \sin 57^\circ = \frac{l}{9.6} \times 9.6 \quad \text{Multiply both sides by 9.6 to isolate } l.$$

$$9.6 \sin 57^\circ = l$$

$$8.05 \approx l$$

Side  $l$  is approximately 8.1 inches long.

b) Use the cosine ratio to solve for  $r$ .

$$\cos P = \frac{\text{adj}}{\text{hyp}}$$

Side  $r$  is approximately \_\_\_\_\_ cm long.

### BUILD YOUR SKILLS

1. Use your calculator or your table to find the following pairs of ratios to four decimal places.

a)  $\cos 23^\circ =$

$\sin 67^\circ =$

b)  $\cos 83^\circ =$

$\sin 7^\circ =$

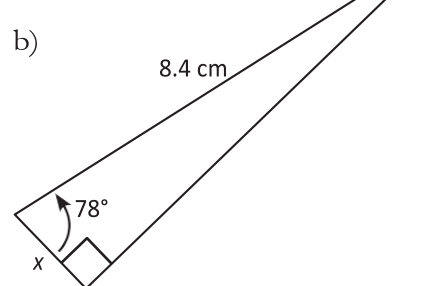
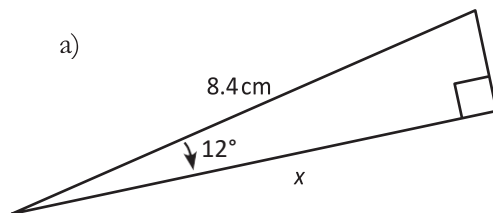
c)  $\cos 45^\circ =$

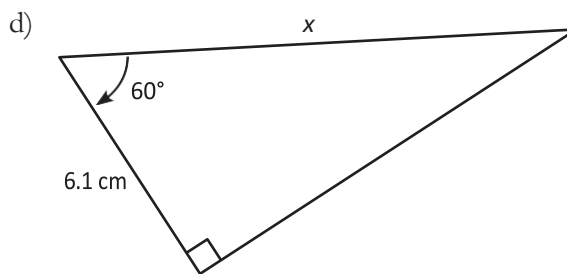
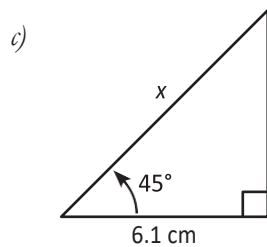
$\sin 45^\circ =$

d)  $\cos 37^\circ =$

$\sin 53^\circ =$

2. Find the measure of the indicated side in each triangle.



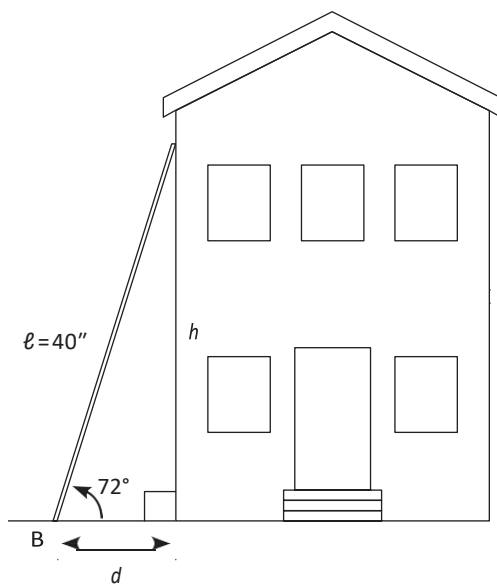


### Example 2

How far from the base of a house is a 40-foot ladder if the angle of elevation is  $72^\circ$ ?

#### SOLUTION

Sketch a diagram.



Use the cosine ratio to solve for  $d$ .

Substitute known values in to ratio  $\cos B = \frac{\text{adj}}{\text{hyp}}$

The ladder rests about \_\_\_\_\_ feet from the house.

**BUILD YOUR SKILLS**

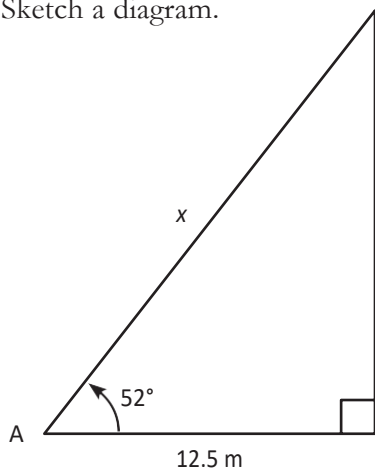
3. How far from the base of a flagpole must a guy wire be fixed if the wire is 12 metres long and it makes an angle of  $63^\circ$  with the ground?
4. Reba walks 25 yards across the diagonal of a rectangular field. If the angle between the width and the diagonal is  $67^\circ$ , how wide is the field?
5. A square pyramid has a slant height of 9 metres. The slant height makes an angle of  $70^\circ$  with the ground. What is the length of a side of the pyramid?

**Example 3**

The angle of a cable from a point 12.5 metres from its base is  $52^\circ$ . How long is the cable?

**SOLUTION**

Sketch a diagram.



Use the cosine ratio to solve for  $x$ , the length of the cable.

$$\cos A = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 52^\circ = \frac{12.5}{x}$$

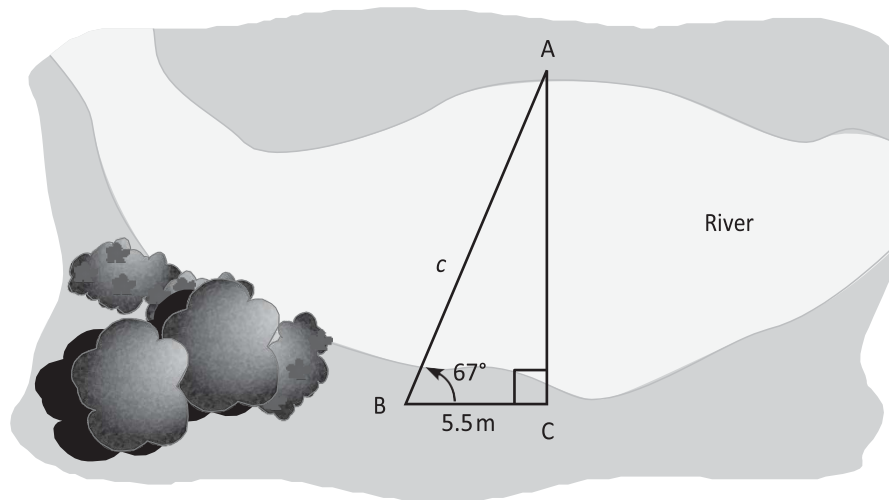
Substitute the known values.  
And calculate

$$x \approx \underline{\hspace{2cm}}$$

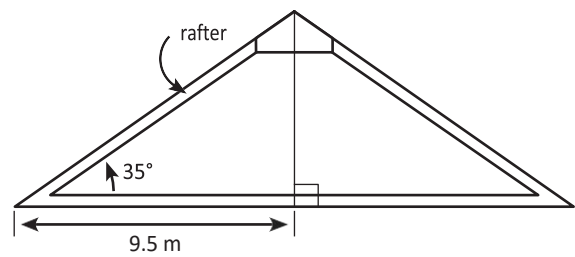
The cable is approximately                                  metres long.

**BUILD YOUR SKILLS**

6. Arul needs to string a bridge line across the river from A to B. What must the length of the bridge line be, given his measurements?



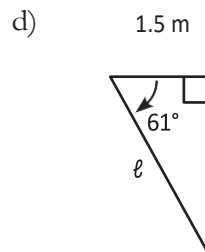
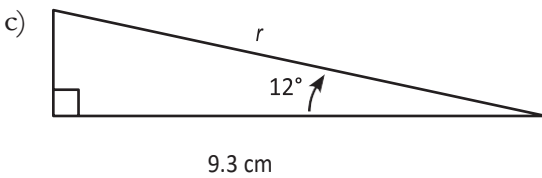
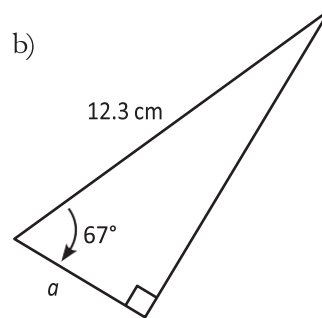
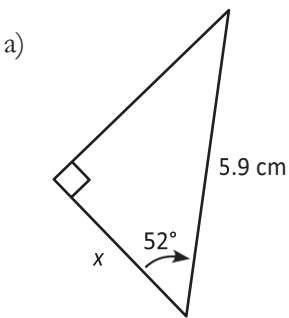
7. What is the length of a rafter that makes an angle of  $35^\circ$  with the floor of an attic whose centre is 9.5 metres from the edge?



8. An airplane starts descending at an angle of depression of  $5^\circ$ . If the horizontal distance to its destination is 500 kilometres, what is the actual distance the airplane will travel before it lands?

### PRACTISE YOUR NEW SKILL

1. Find the lengths of the indicated sides.





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2. A screw conveyor is sometimes used to move grains and other materials up an incline. How far from the base of a barn must a 20-metre screw conveyor be placed if the angle of elevation to the loft is to be  $30^\circ$ ?
  3. What is the slant height of a cone if the diameter is 20 centimetres and the angle made with it is  $65^\circ$ ?
  4. A hot air balloon travels 1.2 kilometres horizontally from its take-off point. The angle of elevation from the take-off point to the balloon is  $15^\circ$ . How far did the balloon travel?

5. What horizontal distance has a car travelled if the incline of the road averages  $3.2^\circ$  and the car's odometer reads 8.5 kilometres?
6. The horizontal distance between two clothesline poles is 3.4 metres. When wet clothes are hung in the middle of the line, it sags at an angle of depression of  $6^\circ$ . How long is the clothesline?