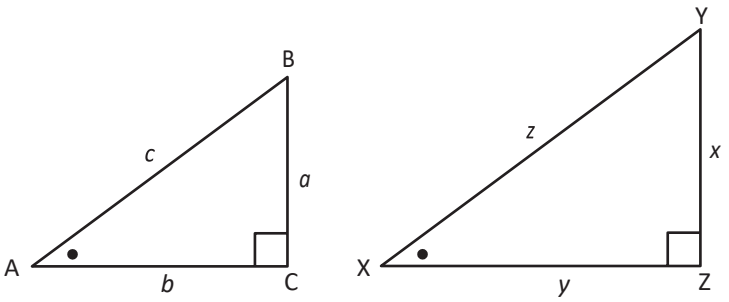
7.2 The Sine Ratio

NEW SKILLS: WORKING WITH THE SINE RATIO TO SOLVE TRIANGLES

In chapter 6, you worked with similar triangles to discover that, in triangles with congruent angles, the ratio between the corresponding sides of the similar triangles is the same.

Angles marked with the same symbol are equal.

The following diagram shows similar triangles. $\Delta ABC \sim \Delta XYZ$.



The ratios between corresponding sides are equal, so we know that the following is true.

$$\frac{a}{x} = \frac{c}{z}$$

This proportion can be rearranged so that each side of the equation represents a ratio of sides from the same triangle.

$$\cancel{x} \times z \times \frac{a}{\cancel{x}} = \frac{c}{\cancel{z}} \times \cancel{z} \times x$$

Multiply both sides by the product of the denominators and simplify.

$$za = cx$$

$$\frac{\cancel{z}\,a}{c\cancel{z}} = \frac{\cancel{c}\,x}{\cancel{c}z}$$

Divide both sides by the same number and simplify.

$$\frac{a}{c} = \frac{x}{z}$$

sine ratio: in a right triangle, the ratio of the length of the side opposite a given angle to the length of the hypotenuse (abbreviated as sin)

When triangles are similar, the ratio of the length of the side opposite a given angle to the length of the hypotenuse is always the same. This ratio is referred to as the **sine ratio**.

Given any right triangle with acute angle A, the sine ratio can be written as follows.

$$\text{sine } \angle A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$$

The ratio is abbreviated as follows.

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

Use your scientific calculator to calculate the values of the sines of angles.

Example 1

Use your calculator to determine the following sine ratios. Round to four decimal places.

a) sin 15° b) sin 30°

c) sin 60° d) sin 80°

What do you notice about these values?

SOLUTION

a) sin 15° =_____ b) sin 30° =_____

c) sin 60° = _____ d) sin 80° =_____

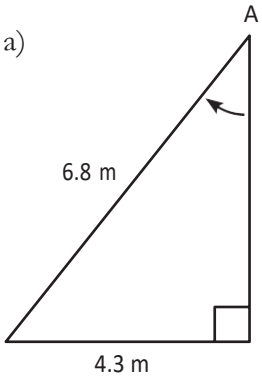
The sine ratio determines that if you have a right triangle with an acute angle given, regardless of the size of the triangle, the ratio of the side opposite that angle to the hypotenuse will always be the same.

The value of the sine ratio increases as the angle gets bigger.

BUILD YOUR SKILLS

1. Calculate the value of sin A to two decimal places.

a)



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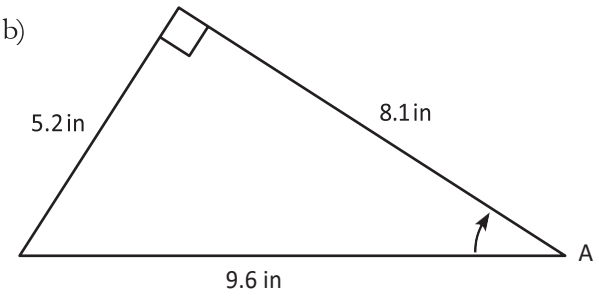


2. Use your calculator to determine the value of each of the following sine ratios to four decimal places.

a) sin 10° b) sin 48°

c) sin 62° d) sin 77°

3. Use your calculator to determine the value of sin 90°. Suggest a reason why this is so.

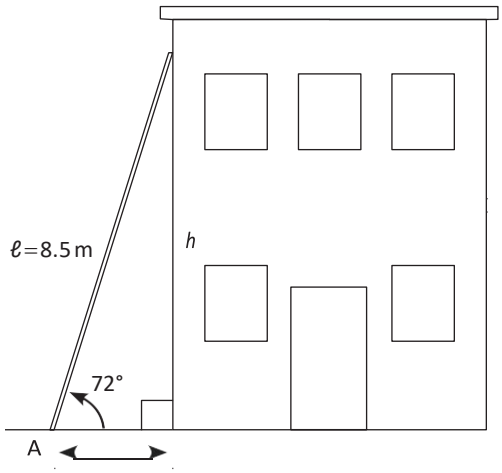
Example 2

The sine ratio can be used to help you find missing parts of a right triangle.

A ladder 8.5 metres long makes an angle of 72° with the ground. How far up the side of a building will it reach?

SOLUTION

Sketch a diagram.



The height, h, is opposite the 72° angle, and the ladder, ℓ, forms the hypotenuse of the triangle. A right triangle is formed, with h as the side opposite the 72° angle, and ℓ as the hypotenuse.

Use the sine ratio to calculate h.

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 72° = \frac{h}{8.5}$$ Substitute the known values.

$$8.5 \times \sin 72° = \frac{h}{8.5} \times 8.5$$ Multiply both sides by 8.5.

$$8.5 \times \sin 72° = h$$

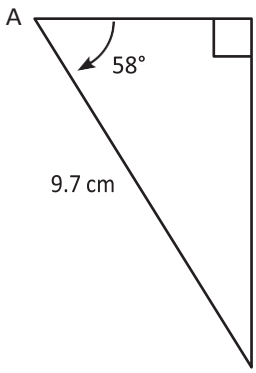
$$8.1 \approx h$$

The ladder reaches approximately 8.1 metres up the side of the building.

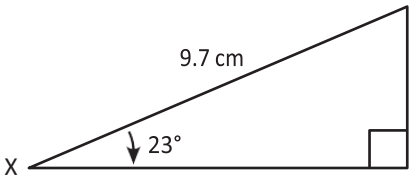
BUILD YOUR SKILLS

4. Calculate the length of the side opposite the indicated angle in the following diagrams.

a)



b)



5. A rafter makes an angle of 28° with the horizontal. If the rafter is 15 feet long, what is the height at the rafter's peak?

6. How high is a weather balloon tied to the ground if it is attached to a 15-metre string and the angle between the string and the ground is 35°?

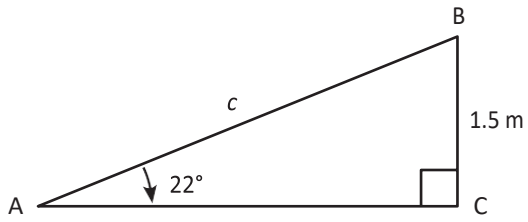
Example 3

Brad is building a ramp. The ramp must form an angle of 22° with the level ground and reach a point that is 1.5 metres above the ground. How long will the ramp be?

Wait until you have isolated the unknown variable before doing the calculation. This will minimize errors due to rounding.

SOLUTION

Sketch a diagram.



Let c represent the length of the ramp. On the diagram, 1.5 metres is opposite the 22° angle.

Use the sine ratio to solve for c.

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 22° = \frac{1.5}{c}$$ Substitute the known values.

$$c \times \sin 22° = \frac{1.5}{c} \times c$$ Multiply both sides by c.

$$c \times \sin 22° = 1.5$$ Simplify.

$$\frac{c \times \sin 22°}{\sin 22°} = \frac{1.5}{\sin 22°}$$ Divide both sides by sin 22° to isolate c.

$$c = \frac{1.5}{\sin 22°}$$

$$c \approx 4.004$$

The ramp is approximately 4 metres long.

Here is an alternative way to calculate this answer.

$0.3746 = \frac{1.5}{c}$

So $1.5 \div 0.3746 = c$

c = 4.004 m

Use simpler numbers

This works because : $4 = \frac{12}{c}$

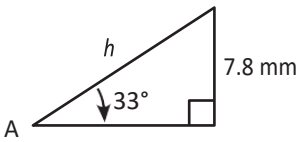
$12 \div 4 = 3$

c =3

BUILD YOUR SKILL

7. Find the length of the hypotenuse in the following diagrams.

a)



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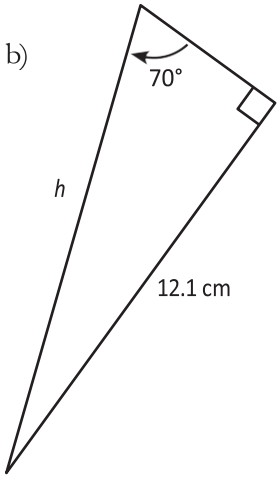


8. How long is a guy wire that is attached 4.2 metres up a pole if it makes an angle of 52° with the ground?

9. A boat is carried with the current at an angle of 43° to the shore. If the river is approximately 15 metres wide, how far does the boat travel before reaching the opposite shore?

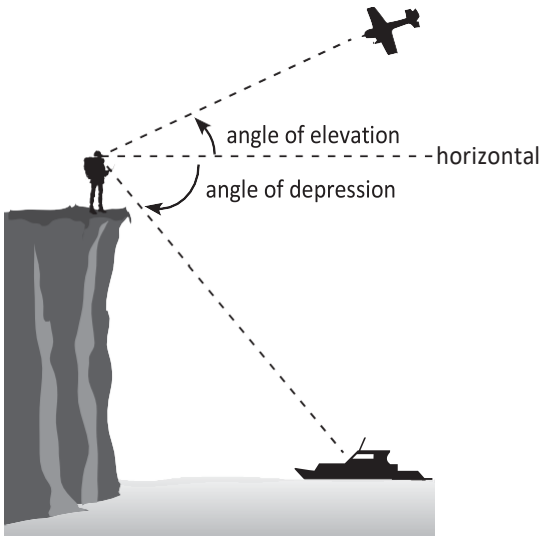
NEW SKILLS: WORKING WITH ANGLE OF ELEVATION AND DEPRESSION

When you look up at an airplane flying overhead, the angle between the horizontal and your line of sight is called an **angle of elevation.** When you look down from a cliff to a boat passing by, the angle between the horizontal and your line of sight is called an **angle of depression**.

angle of elevation: the angle formed between the horizontal and the line of sight while looking upward; sometimes referred to as the angle of inclination

angle of depression: the angle formed between the horizontal and the line of sight when looking downward

For more details, see pages 288–289 of *MathWorks 10.*

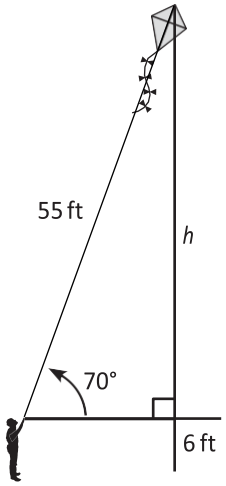


Example 4

The angle of elevation of Sandra's kite string is 70°. If she has let out 55 feet of string, and is holding the string 6 feet above the ground, how high is the kite?

SOLUTION

Sketch and label a diagram.



Use the sine ratio to solve for the height of the kite.

$$\sin H = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 70° = \frac{h}{55} \qquad \text{Substitute the known values.}$$

$$55 \times \sin 70° = \frac{h}{55} \times 55 \qquad \text{Multiply both sides by 55.}$$

$$55 \times \sin 70° = h$$

$$\_\_\_\_\_ \approx h$$

The kite is approximately _____feet above where Sandra is holding it. Add 6 feet for the distance between the ground and the start of the string. The kite is about _____ feet above the ground.

BUILDYOUR SKILL

10. George is in a hot air balloon that is 125 metres high. The angle of elevation from a house below, to the balloon, is 18°. How far is George from the house?

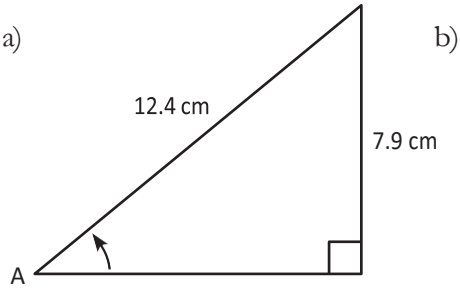
11. The angle of elevation of a road is 4.5°. What is the length of the section of road if it rises 16 metres?

12. The angle of elevation of a slide that is 3.6 metres long is 32°. How high above the ground is the top of the slide?

PRACTISE YOUR NEW SKILL

1. Calculate the sine of the indicated angle.

a)



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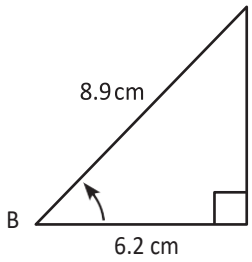
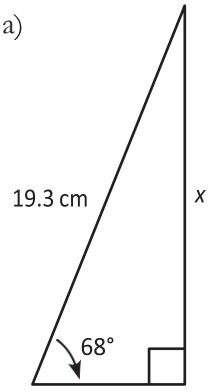


2. Calculate the length of the indicated side.

a)



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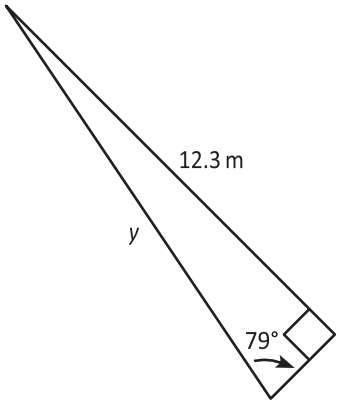

3. A ramp with a length of 21.2 metres has an angle of elevation of 15°. How high up does it reach?

4. The angle of elevation from the bottom of a waterslide to the platform above is 20°. If the waterslide is 25 metres long, how high is the platform?

5. A man walks at an angle of 68° north of east for 45 metres. How far north of his starting point is he?