## Chapter 7 Trigonometry of Right Triangles



This man is installing an angle brace. By incorporating right triangles, angle braces provide support and strength to bookshelves, building roofs, and beyond.

## 7.1

## The Pythagorean Theorem

## REVIEW: WORKING WITH TRIANGLES

Each vertex of a triangle is labelled with an upper case letter, and each side is labelled either with the lower case letter corresponding to the opposite vertex or with the upper case letters of the vertices it connects.

## Example 1

Consider $\triangle R S T$.

a) Label the sides with the appropriate lower case letter.
b) Name the sides using the upper case letters of the vertices they connect.

## SOLUTION

a) Each side is labelled with the lower case letter corresponding to the opposite vertex.

s
b) The sides can also be named according to the upper case letters of the vertices they connect.

Side $r$ can be called side $\qquad$ .

Side $s$ can be called side $\qquad$ .

Side $t$ can be called side $\qquad$ .

## BUILD YOUR SKILLS

1. Label each side of the triangles below using a single lower case letter corresponding to the opposite vertex.
a)

b)

c)

d)

2. Label each vertex of the triangles below using a single upper case letter corresponding to the opposite side.
a)

b)


d)


A right triangle is a triangle with one right angle. The side opposite the right angle is the longest side and is called the hypotenuse. The other two sides are called legs (or, in some cases, arms).

The Pythagorean theorem states the relationship among the sides of a right triangle.
Given a right triangle ABC with right angle C , the Pythagorean theorem states the following.
$a^{2}+b^{2}=c^{2}$
For more details, see page 272 of MathWorks 10.

## Example 2

Label the sides of the triangles and state the Pythagorean theorem as it applies to them.


SOLUTION


Since $\qquad$ is the hypotenuse, the Pythagorean theorem is written as follows.

$$
p^{2}+r^{2}=q^{2}
$$


hypotenuse: the longest side of a right triangle, opposite the $90^{\circ}$ angle

Pythagorean theorem: in a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse

## BUILD YOUR SKILL

3. Given the following diagram, use the lettering provided to state three Pythagorean relations that apply.

4. A ladder, $\ell$, is placed against the side of a house, $h$. The foot of the ladder is a distance $d$ from the base of the house. Draw a diagram and express the relationship that exists between $\ell, h$, and $d$.
5. Rearrange the Pythagorean theorem to solve first for $x$ and then for $y$.

$$
x^{2}+y^{2}=z^{2}
$$

## Example 3

Use the Pythagorean theorem to find the lengths of the missing sides of the triangles to the nearest tenth of a unit.
a)
P

b)
Y


SOLUTION
a) Write the Pythagorean theorem using the labels on the given triangle.

$$
\begin{aligned}
p^{2}+r^{2} & =q^{2} \\
5.2^{2}+3.8^{2} & =q^{2} \quad \text { Substitute the known values. } \\
27.04+14.44 & =q^{2} \\
& =q^{2} \\
& =q \quad \text { Take the square root of both sides. } \\
& \approx q
\end{aligned}
$$

Side $q$ is approximately $\qquad$ m.
b) Write the Pythagorean theorem using the labels on the giventriangle.

$$
y^{2}+z^{2}=x^{2}
$$

## BUILD YOUR SKILL

6. Calculate the values of $x$ and $y$.

7. A 40 -foot ladder reaches 38 feet up the side of a house. How far from the base of the house is the foot of the ladder?
8. A field is 120 m by 180 m . How much shorter is your route if you walk diagonally across the field rather than walking around the edge to the opposite corner?

## PRACTISE YOUR NEW SKILLS

1. A stairway rises 6 feet 4 inches over a horizontal distance of 8 feet 6 inches. What is the diagonal length of the stairway?
2. A 28 -metre long guy wire is attached to a point 24 m up the side of a tower. How far from the base of the tower is the guy wire attached?
3. The construction plans for a ramp show that it rises 3.5 metres over a horizontal distance of 10.5 metres. How long will the ramp surface be?
4. The advertised size of a TV screen is the distance between opposite corners. Sally bought a 52 -inch TV. If the height of the TV is 32 inches, how wide is it?
5. A boat sailed due north at a rate of $12 \mathrm{~km} / \mathrm{h}$ for 3 hours, then due east at a rate of $18 \mathrm{~km} / \mathrm{h}$ for 2 hours. How far was it from its starting point, measuring the shortest distance?
