Chapter 10.4 - 10.7

Speed–Time Graphs for Acceleration

Scientists use many different ways of communicating what they observe: some methods are simple (such as a straightforward observation); some are more comprehensive (such as a graph); and others are more powerful (such as a scientific law). Knowing and using a variety of ways of communicating helps us to develop a better understanding of a topic such as acceleration.

Acceleration is a description of the relationship between speed and time. In the previous section we described acceleration using words, a number with units, and a mathematical definition. In this section we will use tables and graphs to communicate acceleration.

Acceleration is basically a change in speed over time. **Figures 1** and **2** show speed increasing or decreasing as a function of time. As you know, the slope of a line on a graph indicates the rate of change in one variable (Δy) compared to a second variable (Δx). If the variables are speed (on the y-axis) and time (on the x-axis), then the slope ($\Delta y/\Delta x$) corresponds to the mathematical definition of acceleration ($\Delta v/\Delta t$). The units of the slope of a speed–time graph are the units of speed divided by the units of time. For example, in **Figure 3**

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$

Acceleration of a Snowboarder

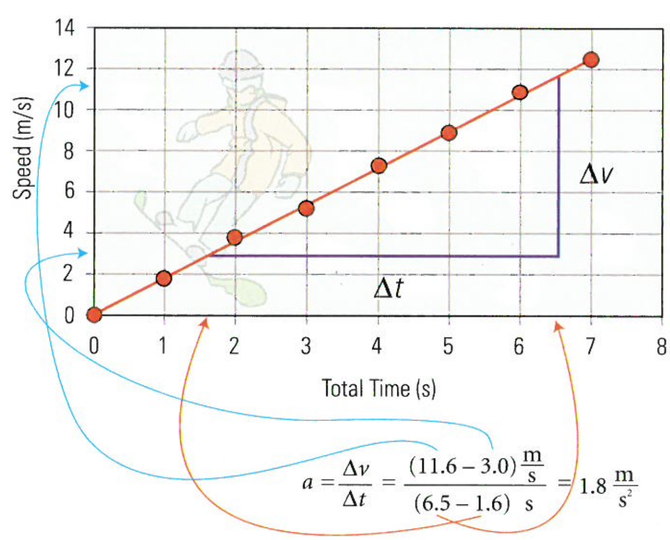


Figure 3
Acceleration of a snowboarder

Acceleration — Speeding Up

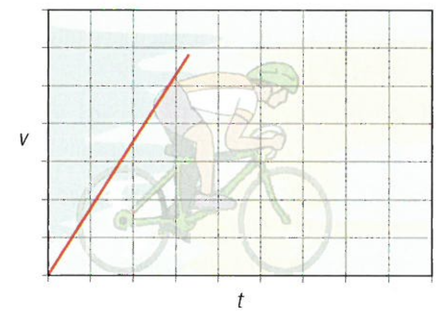


Figure 1
A cyclist accelerates at the start of a race.

Acceleration — Slowing to a Stop

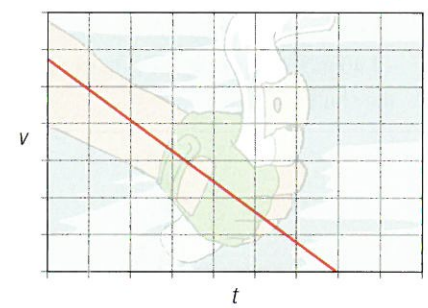


Figure 2
Braking results in negative acceleration.

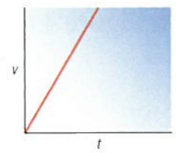
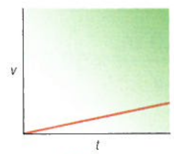
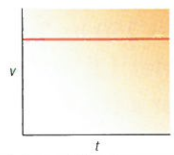
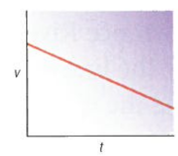
Did You Know?

Nicole Oresme (c. 1323–1382), working at the University of Paris, developed what we now know as graphing for motion—perhaps the first graphing. He plotted speed against time for constant acceleration.

The type of slope of a speed–time graph tells us a lot about the type of acceleration (**Table 1**): a positive slope represents positive acceleration (an object speeding up); a negative slope represents negative acceleration (an object slowing down). In addition, the steepness of the slope represents the size of the acceleration.

In summary, the **slope of the line** on a speed-time graph provides the acceleration. A positive slope represents positive acceleration (object speeding up) and a negative slope represents negative acceleration (object slowing down). Now let's consider the area under the line on a speed-time graph. What does it give us?

Table 1 **Relationship Between Slope and Acceleration**

Slope	Sample Speed–Time Graph	Interpretation
high positive value		high positive acceleration (rapidly increasing speed)
low positive value		low positive acceleration (slowly increasing speed)
zero		zero acceleration (constant speed)
negative value		moderate negative acceleration (decreasing speed)

Area Under the Line on a Speed–Time Graph

Suppose you are training for a bicycle race and your racing partner records your speed every 10 s over a total distance of 250 m (**Table 2**). Notice that the speed increases by 2.0 m/s every 10.0 s. This represents a constant acceleration because the increase in speed, Δv, is the same for every equal time interval, Δt. The speed–time graph (**Figure 4**) illustrates constant acceleration. You know that the slope of the line on a speed–time graph represents the acceleration. What feature, if any, of this graph would give us the distance travelled: 250 m? Notice that the distance units (m) can be obtained from multiplying speed (m/s) by time (s).

Table 2 **Acceleration on a Bicycle**

Time (s)	Speed (m/s)
0.0	0.0
10.0	2.0
20.0	4.0
30.0	6.0
40.0	8.0
50.0	10.0

For example,

$$1\text{ m} = 1\,\frac{\text{m}}{\cancel{\text{s}}} \times 1\,\cancel{\text{s}}$$

This also corresponds to the distance as calculated from the defining equation for speed:

$$v = \frac{\Delta d}{\Delta t} \qquad \Delta d = v\Delta t$$

Two variables multiplied together suggest the area of a geometric shape. Is there an area corresponding to 250 m in the speed–time graph in **Figure 4**? The area of the rectangle bounded by the axes and dotted lines would be 500 m.

$$\begin{aligned} \text{area} &= 10\,\frac{\text{m}}{\cancel{\text{s}}} \times 50\,\cancel{\text{s}} \\ &= 500\text{ m} \end{aligned}$$

The distance travelled is actually 250 m or one-half of this area. The area of the triangle underneath the line on the graph is one-half the area of the complete rectangle. Does the area of this triangle relate to the distance travelled?

We have no proof, but rather a mathematical theorem at this point. Testing this theorem in the laboratory clearly shows that **the area under the line in a speed–time graph equals the distance travelled during the time interval.**

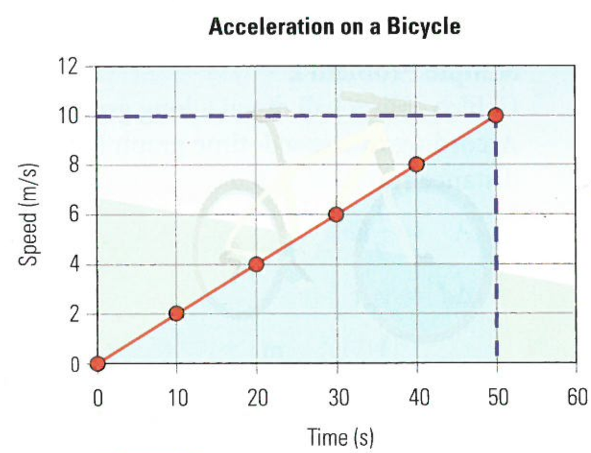


Figure 4
How is the speed changing with time?

For constant acceleration, starting from rest, the area under the line will always be the area of a triangle. From your experience in mathematics, you know that the area of the triangle is one-half of the base times the height of the triangle. From **Figure 4** the distance travelled is calculated as follows:

$$A = \frac{1}{2}hb \quad \text{or} \quad (area = \frac{1}{2} \times height \times base)$$

$$\Delta d = v_{av}\Delta t$$

$$= \frac{1}{2} \times \frac{10.0 \text{ m}}{\cancel{\text{s}}} \times 50.0 \cancel{\text{s}}$$

$$= 250 \text{ m}$$

As indicated by the area under the line, the distance travelled during the 50.0 s time interval is 250 m. You will study areas and distances travelled in more detail in Chapter 12. At this point we will restrict ourselves to the area of a rectangle or a triangle.

Sample Problem 1

A boat on the St. Lawrence River travels at full throttle for 1.5 h. From the area under the line of the speed–time graph (**Figure 5**), determine the distance travelled.

$v = 30$ km/h

$t = 1.5$ h

$\Delta d = ?$

$$A = wl$$

$$\Delta d = 30 \frac{\text{km}}{\cancel{\text{h}}} \times 1.5 \cancel{\text{h}}$$

$$= 45 \text{ km}$$

Based upon the area under the line of the graph, the distance travelled by the boat is 45 km.

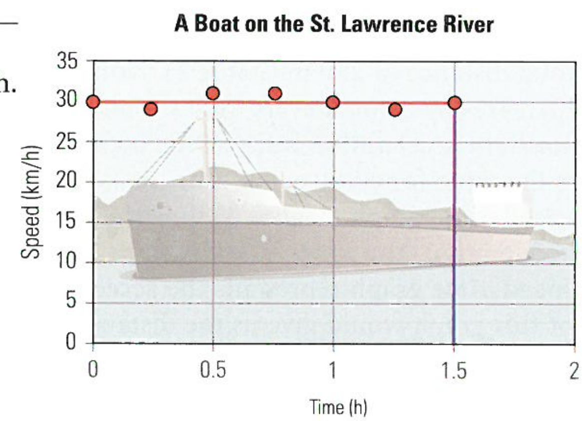


Figure 5

Speed–time graph for a boat

Note that, in Sample Problem 1, the speed is a fairly constant 30 km/h. This is also the average speed.

Let's now look at an example of a uniformly accelerating object.

Sample Problem 2

Galileo rolls a ball down a long grooved inclined plane. According to a speed–time graph (**Figure 6**), what is the distance travelled in 6.0 s?

$$A = \frac{1}{2}hb$$

$$\Delta d = v_{av}t$$

$$\Delta d = \frac{1}{2} \times 7.2 \frac{\text{m}}{\cancel{\text{s}}} \times 6.0 \cancel{\text{s}}$$

$$= 22 \text{ m}$$

Based upon the area under the line of the graph, the distance travelled by the ball in 6.0 s is 22 m.

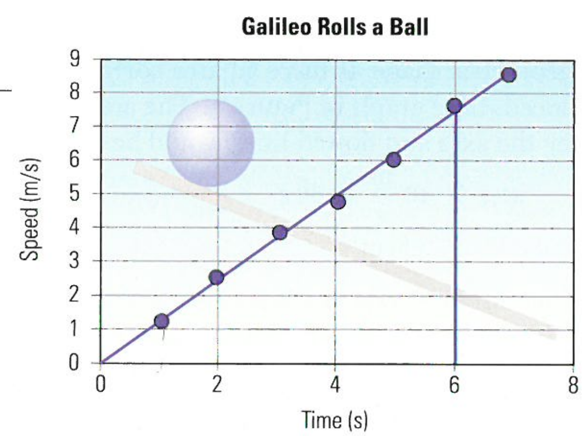


Figure 6

Speed–time graph for a ball

Understanding Concepts

1. How can you tell from a speed–time table whether an object is accelerating?
2. How can you tell from a speed–time graph whether an object is accelerating?
3. Sketch a speed–time graph with two separate labelled lines for
   (a) high positive acceleration;
   (b) low negative acceleration.
4. What feature of a speed–time graph communicates
   (a) the acceleration?
   (b) the distance travelled?
5. Two runners, Cathryn and Keir, take part in a fundraising marathon. The graph in **Figure 7** shows how their speeds change for the first 100 s from the start of the marathon.

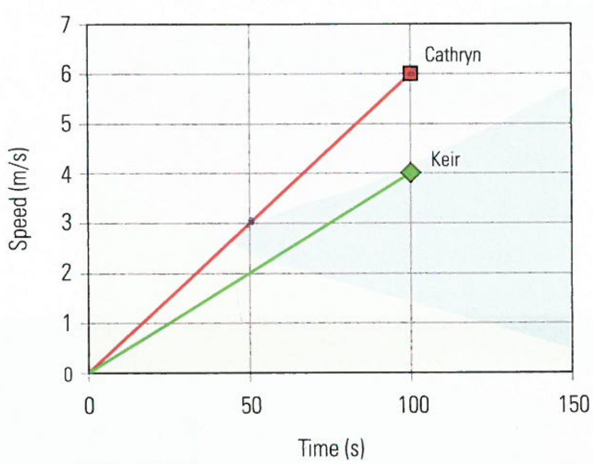


Figure 7

   (a) Which runner has the greater acceleration? Show this by calculating the acceleration of each.
   (b) Which runner is ahead after 100 s? Calculate and compare the distance travelled by each.
6. The cheetah is the fastest land animal and can accelerate rapidly in an attack. **Table 3** shows some typical speeds and times for a cheetah.
   (a) Draw a speed–time graph using the information in **Table 3**.
   (b) Using your graph, calculate the average acceleration of the cheetah.
   (c) Using your graph, calculate the total distance travelled by the cheetah by the end of 2.0 s.

Table 3 **Acceleration of Cheetah**

Time (s)	Speed (m/s)
0.0	0.0
0.5	5.0
1.0	10.0
1.5	15.0
2.0	20.0

7. Create a scientific question about the acceleration characteristics of different vehicles. State the variables clearly.
8. Sketch and label distance–time and speed–time graphs for constant speed and a speed–time graph for constant acceleration (three graphs in total).
9. Why does $\Delta d = v_{av}\Delta t$ but $A = \frac{1}{2}hb$? Where does the half (1/2) come from? If $\Delta d = A$ and $\Delta t = b$, then why does $v_{av} = \frac{1}{2}h$?
10. Draw a speed–time graph for your movements as you go from your desk in the classroom to the pencil sharpener.
11. Clayton sets out on his motorcycle. His speed at different times is shown on the graph in **Figure 8**.

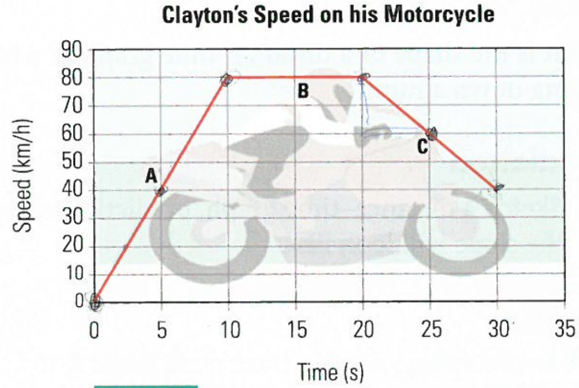


Figure 8

   (a) Calculate the accelerations during each of the time intervals, A, B, and C.
   (b) Without calculating, list the time intervals during which the distances travelled are, in order, from largest to smallest.

Reflecting

12. What assumption have you been making about acceleration in this chapter?

SKILLS HANDBOOK: (V) Graphing (K2) Asking the Questions

10.5 Investigation

INQUIRY SKILLS MENU

- ○ Questioning
- ○ Hypothesizing
- ● Predicting
- ● Planning
- ● Conducting
- ● Recording
- ● Analyzing
- ● Evaluating
- ● Communicating

Graphing Distances During Acceleration

In all sciences, especially physics, technology is crucial to scientific investigations. Technology is particularly valuable for collecting and analyzing evidence from an experiment of motion (**Figure 1**). Many technological instruments initially depended on scientific research for their invention, and then science was advanced by the use of these instruments. Laser photogates (**Figure 2**) and spark timers are two examples of this interplay between science and technology. In this experiment you will use very simple technologies—a stopwatch and a ruler—to discover the relationship between distance travelled and time when an object is accelerating. The results obtained will not be as precise or as certain as they would be if we used other, more modern, technologies. This is an important point to realize when evaluating an experiment.

The purpose of this investigation is to determine the shape of a distance–time graph for acceleration and to practise evaluating experiments.

Figure 1
As the airplane accelerates, it travels a greater distance during each time period.

Question

What is the shape of a distance–time graph of a ball rolling down a ramp?

Prediction

(a) Sketch a distance–time graph, predicting what (V2) the slope will look like.

Design

Roll a solid sphere down a track from point A to point B (**Figure 3**). During the investigation, alter the distance between A and B (the independent variable) from about 25 cm to 150 cm. For each distance, measure time (the dependent variable) using a stopwatch. Use the same ball, track, and slope of track (controlled variables) for all measurements.

(b) Prepare a labelled table in which to record your (K7) evidence. You will need to record both the independent and the dependent variables.

(c) Using the information presented above, write (K6) out the numbered steps that you will follow to carry out this experiment. Include all necessary safety precautions.

Figure 2
A laser photogate detects the time when an object crosses the beam between the bottom ends of the U-bracket.

SKILLS HANDBOOK: (V2) Graphing (K7) Preparing Tables of Evidence (K6) Writing a Procedure

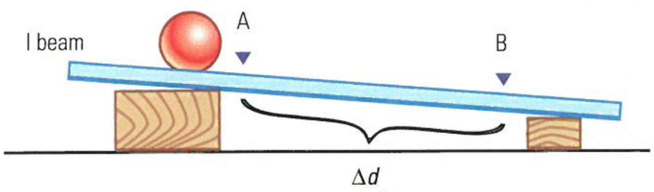


Figure 3
A ball rolls down the I-beam from point A to point B.

Materials

- I-beam track and supports (about 2 m long)
- large wood or steel ball
- metre stick or metric tape
- stopwatch

Procedure

(d) When your teacher has approved your procedure, carry it out and record your observations.

Analysis and Evaluation

(e) Construct a graph of distance versus time and answer the question. Use the *y*-axis for distance and the *x*-axis for time. (V2)

(f) Compare your graph from (e) with your prediction in (a). Account for any differences by explaining what you have learned in this investigation.

(g) Evaluate the experimental design, materials, procedure, and your skills. Note any flaws, and suggest improvements for this experiment. Identify and describe sources of experimental uncertainty. (O)

(h) Share your procedure by meeting with other groups or as part of a class discussion. Note what others have done that your group did not. After this discussion you should be able to improve your evaluation of this experiment.

Understanding Concepts

1. Suggest another experiment related to this one. Using clearly stated variables, write the question that would be answered by the related experiment. (K2)
2. Repeating experiments is an important part of the scientific process. Why is it important?

Exploring

3. **(a)** Predict how your results would change if the ramp were more steeply sloped.
   (b) What difficulties would this steep ramp introduce into the experimental procedure?

Reflecting

4. Generally, when creating graphs, we place the independent variable on the *x*-axis. What, do you suppose, is the reason for breaking this rule when we draw speed–time and distance–time graphs?

Challenge

2 Galileo used an experimental set-up similar to that shown in **Figure 3**. Why?

(O) Evaluating (K2) Asking the Question

10.6 Case Study

Buying a Car?

When people in Western countries travel, they use mostly cars. Advertising about cars may focus on beauty, lifestyle, image, aerodynamics, fuel economy, safety, or acceleration (**Figure 1**). Many magazines are dedicated to discussions of automobiles and their characteristics (**Figures 2** and **3**). The buying, selling, and advertising of cars occupies significant space in daily newspapers, and time on radio and TV stations. In short, our society seems to be preoccupied with the automobile.

(a) In general, how much of all car advertising would you consider to be practical (related to useful performance and safety) and how much is devoted to all other aspects of the car?

(b) If you were going out to buy a car tomorrow, what would be your top three expectations for your ideal car?

Many potential buyers read road-test information. The purpose of road tests is to compare different vehicles and to give an idea of how each car will perform. To understanding the results of these tests, you need to understand the basic concepts of the physics of motion.

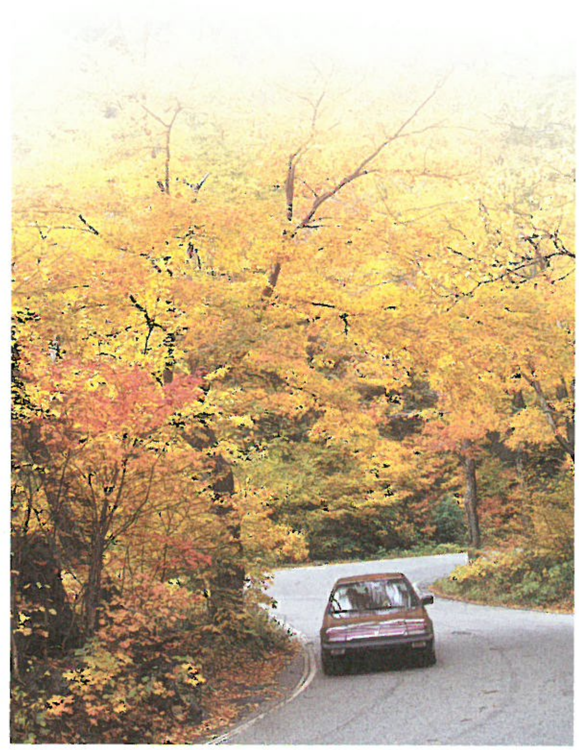


Figure 1
The performance of a car is important for enjoyment as well as for safety reasons.

Figure 2
Professional drivers test cars under carefully controlled conditions.

Figure 3
Testing the performance of a car involves taking many measurements with high-tech equipment.

Car Road Test

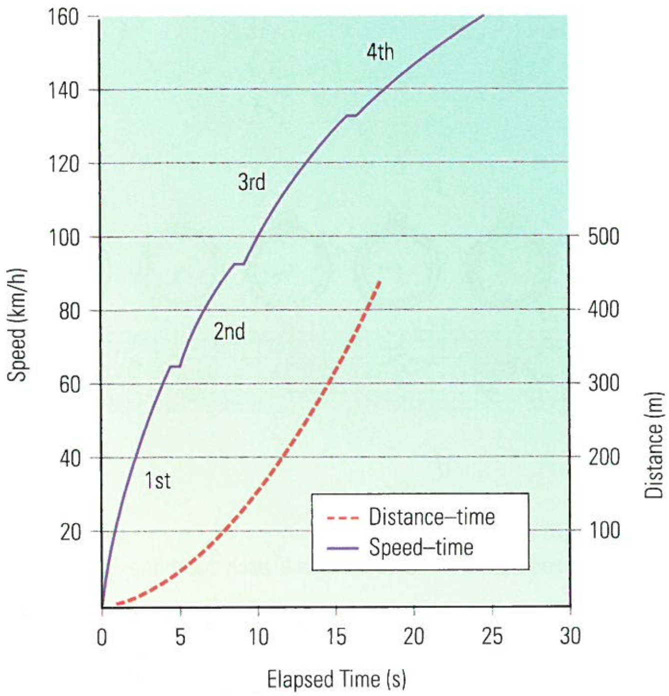


Figure 4

A standard road test provides speed–time and distance–time graphs. In this case the graphs are plotted on the same time axis.

Figure 4 displays some of the typical road test information in graphical form. The speed–time graph provides the speed during a 25-s time interval and the distance travelled during an 18-s time interval. Car magazines typically plot the speed and distance graphs on a common time (*x*-) axis. The curved shapes of both lines demonstrate acceleration. In this figure the speed is plotted as the car accelerates from the first gear on through the fourth gear. You can see on the graph where the driver shifted gears.

(c) According to the speed–time graph in the road test (**Figure 4**), in what gear is the average acceleration of the car the greatest? In what gear is it the least?

(d) What happens in the car at the small level sections that divide the speed–time curve into four parts?

(e) Calculate the average acceleration of the vehicle in (km/h)/s during the first 15 s.

(f) According to **Figure 4**, what is the approximate distance travelled in the first 10.0 s?

Table 1 presents the braking distances for the same car to come to rest from the specified speed.

Braking distance is a performance standard that relates to safety. There are, of course, other safety standards that can be compared from car to car.

Table 1 Braking Distances

Speed (km/h)	Distance (m)
100	51
130	91

(g) How does the braking distance at 100 km/h compare with the braking distance at 130 km/h (**Table 1**)? What does this suggest about the safe distance between cars travelling in the same direction along a road?

(h) Braking distance includes only the distance travelled once the brakes are applied. What other factor is important once you realize that you have to stop quickly?

If you are trying to decide among different models and brands, road tests provide quantitative information in a standard way. We can use the information to compare the performance of various vehicles.

(i) If you were comparing cars for possible purchase, what information in a standard road test would interest you the most? Why?

(j) What important consumer information is not given in the standard road test?

Understanding Concepts

1. Picture two vehicles with the same change in speed, Δv. If the elapsed time for one (A) is half the value for the other (B), how do their average accelerations compare?

Exploring

2. (K4) Outline a procedure for doing a road test similar to **Figure 4** for a bicycle with gears. Plan to obtain the same kind of information as car road tests but using a scale suitable for bicycles. Decide on your independent, dependent, and controlled variables. Assume that you have access to a stopwatch and other required materials.

Challenge

3 From a distance–speed graph, how would you determine the braking distance for a variety of speeds?

10.7

Instantaneous Speed

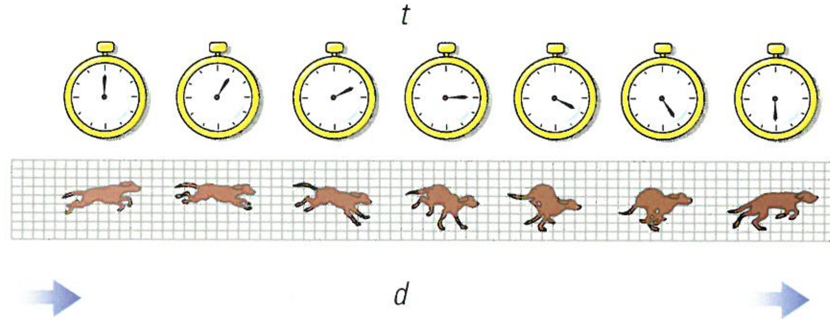


Figure 1
A dog running at a constant speed covers the same distance in each equal time interval.

When you glance at the speedometer while travelling in a car, what you see is the speed at that particular instant. Whether this was the speed earlier in the trip, or not, does not affect the current reading. **Instantaneous speed** is the speed at a particular moment in time.

The dog in **Figure 1** is covering the same distance in each time interval. If you take the pictures in rapid succession, with a shorter time interval, the dog will still travel equal distances for each time interval. The dog is moving at a uniform or constant speed. **Figure 2** shows the same information in the form of a distance–time graph. The ratio of the distance travelled to the time interval over which this occurs is the slope. Because the speed is constant, the slope is constant. On a speed–time graph (**Figure 3**), a constant speed is shown as a horizontal line. The speed at any instant of time, such as 1 s, 2 s, or 3 s, is always the same.

For any object moving at a constant speed, the instantaneous speed is the same at any time, and equals the constant speed.

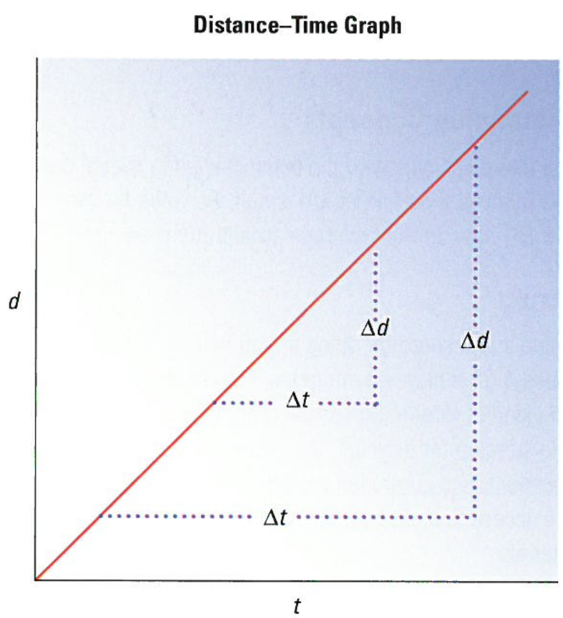


Figure 2
The slope of a distance–time graph represents the speed. The slopes at different times are always the same if the speed is constant.

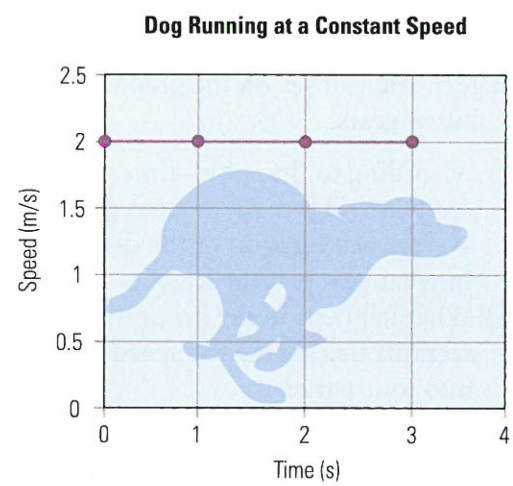


Figure 3
The instantaneous speed at 1 s, 2 s, and 3 s is the same value as at any other time.

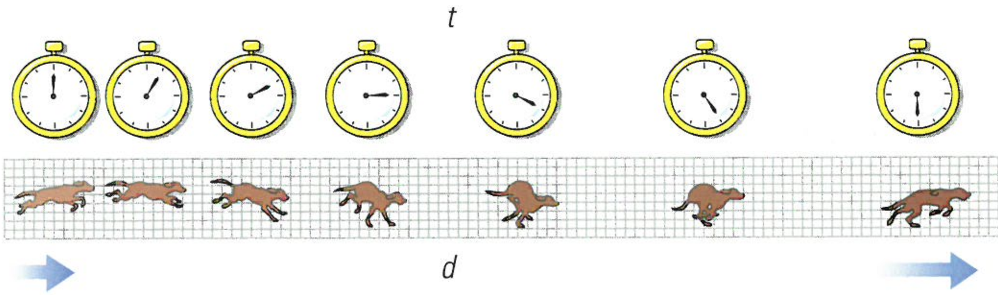


Figure 4

A dog is accelerating and covering greater distances in each successive time interval. The distance–time ratio increases.

Now suppose that the dog sees something interesting and accelerates to investigate. Notice that the distance travelled in each successive time interval increases (**Figure 4**). This means that the ratio of Δd to Δt, or the speed, increases (**Figure 5(a)**). However, if you want an accurate instantaneous speed from the distance–time graph, you need to make the time interval very small. Mathematically, this is accomplished by drawing a tangent to the curve at a particular instant of time. A **tangent** is a straight line that just touches a curve at one point, and represents the instantaneous slope of the line at that point. Notice in **Figure 5(b)** how the slopes of the tangents increase along the curve. Every tangent that you could possibly draw has a different slope and each value of the slope produces a point on the speed–time graph (**Figure 5(c)**).

- On a distance–time graph, the instantaneous speed is the slope of a tangent to the curve at that moment.
- On a speed–time graph, the instantaneous speed is read directly from the line on the graph for that moment.

Did You Know?

A series of definitions of speed, instantaneous speed, non-uniform speed, and constant acceleration was created by a group of mathematicians from Merton College, England, from 1325 to 1350. They are known in history as the Merton Group.

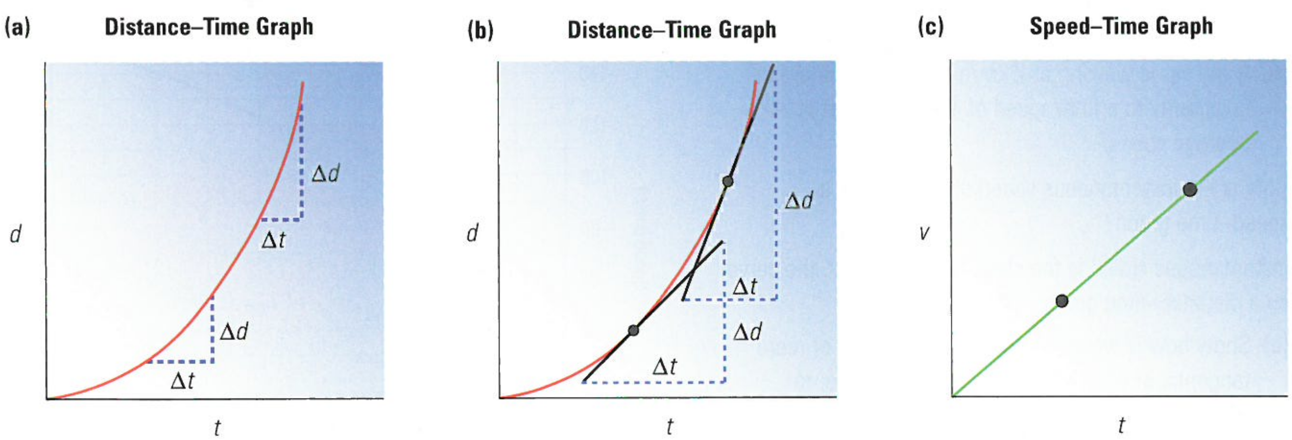


Figure 5

(a) A slope calculation on a curved line can only be approximate because the graph's line is not straight.
(b) A tangent estimated at a point along a curve represents the slope of the line at that point in time. The slope of the line is, of course, the speed at that moment—the instantaneous speed.
(c) It is much easier to read the instantaneous speed at different times on a speed–time graph than on a distance–time graph.

Average Speed

For the dog running at a constant speed, the average speed is the same as the constant speed (**Figure 3**). When the dog is accelerating, the speed is continually changing (**Figure 5**). What is the average speed for the accelerating dog? You can calculate this as you did before by dividing the total distance travelled by the total time.

$$v_{av} = \frac{\Delta d}{\Delta t}$$

Alternatively, you can find average speed from a speed-time graph (**Figure 6**).

Classifying speed as constant, non-constant (changing), instantaneous, or average is useful for organizing and presenting our knowledge about the motion of an object.

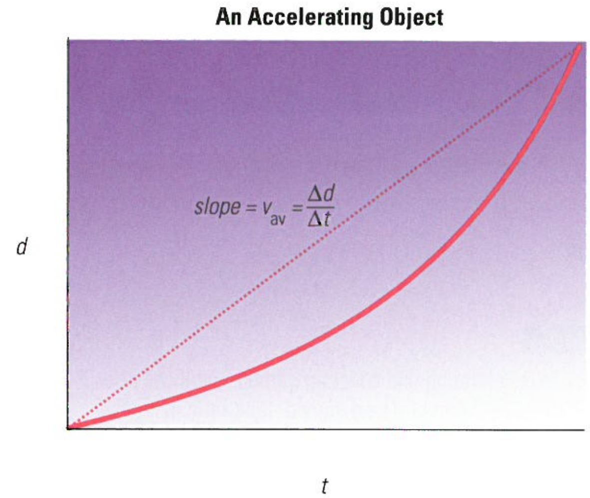


Figure 6
Whatever the curve we find the average speed from the slope of the straight line joining two points on the curve.

Understanding Concepts

1. State, in your own words, the meaning of constant speed, average speed, and instantaneous speed.
2. **(a)** Describe the situation (for example, for the dog in **Figures 1** and **4**) when the instantaneous speed and the average speed are always the same.
   (b) Describe the situation when the instantaneous speed is equal to the average speed at only one moment in time.
3. **(a)** If a cat is running at a constant speed of 10 km/h for 5 s, what is its average speed and what is its instantaneous speed at 4 s?
   (b) If the cat is walking at 2 km/h and then accelerates constantly to a final speed of 14 km/h, what is its average speed?
4. How is the instantaneous speed obtained from a speed–time graph?
5. Instantaneous speed is the slope of a tangent to the curve on a distance–time graph.
   (a) Show how it would be possible to use two or more tangents, at different points along the curve, to calculate acceleration.
   (b) What error might there be in this situation?
6. The performance of a new Mercury Cougar is determined on a test track (**Figure 7**). The car covered a total distance of 402 m during the 16-s time interval.
   (a) During which time interval(s) is the acceleration approximately constant?
   (b) What is the instantaneous speed at 2 s, 8 s, and 14 s?
   (c) What is the average speed for the whole road test? (Note that the acceleration is not constant throughout the test.)

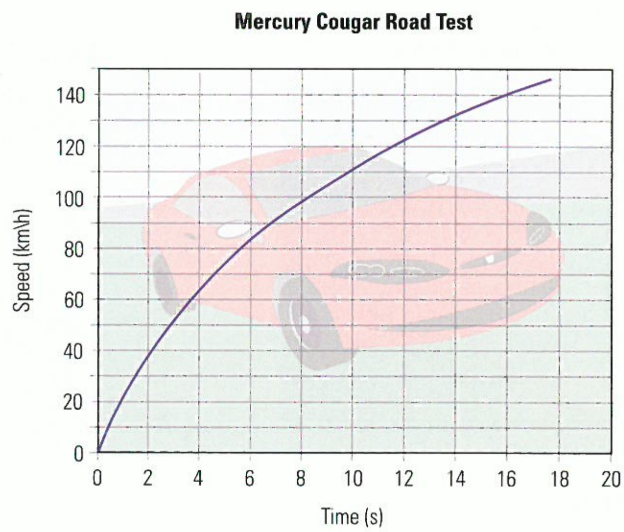


Figure 7

7. **Figure 8** shows three different bicycle trips labelled A, B, and C.

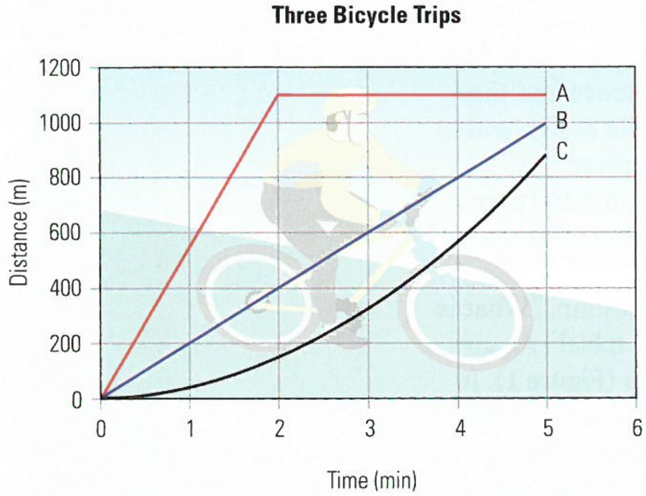


Figure 8

(a) Which graph illustrates a constant speed for the whole trip?

(b) Which graph shows a constantly changing speed?

(c) Which graph(s) has (have) an instantaneous speed of zero at some point?

(d) What is the instantaneous speed at 1.0 min for each graph?

(e) Calculate the average speed for 0 to 5.0 min for each cyclist.

8. When a police officer uses a radar gun to measure a vehicle's speed, what type of speed is measured?

9. Sketch three graphs on one set of axes to illustrate what happens to a distance–graph at medium, low, and and high accelerations.

Making Connections

10. A photograph of a hummingbird may show the bird's body very clearly but the wings are often blurred (**Figure 9**). What does this suggest about the instantaneous speed of the bird's wings and why the photograph is blurred? Using your knowledge of motion and specifically distance, speed, and time, state what is required to get a clear photograph of the bird's wings.

Figure 9

Exploring

11. Describe a common situation in which the same instantaneous speed occurs at least twice but the object is not travelling at a constant speed.

Challenge

3 Which kind of speed (instantaneous or average) is a more important consideration for safe driving? Why? What are some sources of information about this speed, for a driver?